(Time: 3 Hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ [5]

(b) Find k such that f (z) = $\frac{1}{2}$ log ($x^2 + y^2$) + i $tan^{-1}\frac{kx}{y}$ is analytic [5]

(c) Calculate the Spearman's rank correlation coefficient R [5]

X : 10 , 12 , 18 , 18 , 15 , 40 .

Y : 12 , 18, 25 , 25 , 50 , 25 .

(d) Find the inverse Laplace transform of $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$. [5]

Q2.

(a) A continuous random variable has probability density function

f (x) = k ($x - x^2$) , $0 \leq x \leq 1$.

f (x) = 0 otherwise

Find k , mean and variance. [6]

(b) Find the Laplace transform of $e^{-3t} \int_0^t u \sin 3u \, du$. [6]

(c) Obtain the Fourier series to represent f (x) = x^2 in (0, 2π)

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ ..... [8]

Q3.

(a) If the imaginary part of the analytic function w = u + i v = f (z) is [6]

V = $x^2 - y^2 + \frac{x}{x^2 + y^2}$, then show that u = - 2 x y + $\frac{y}{x^2 + y^2}$.

(b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$ [6]

(c) Fit a second-degree parabolic curve and estimate y when x = 10

x : 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 ,

y : 2 , 6 , 7 , 8 , 10 , 11 , 11 , 10 , 9 . [8]

Q4.

(a) Obtain the Fourier series to represent f (x) = x^3 in ($-\pi, \pi$). [6]

(b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for the following data

X : 65 , 66 , 67 , 67, 68 , 69 , 70 , 72 .

Y : 67 , 68 , 65 , 66 , 72 , 72 , 69 , 71 . [6]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t}$ dt = $\frac{\pi}{8}$. [8]

Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$. [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P X = x)	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments . [6]

(c) Obtain the half range cosine series of $f(x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4}$ …….. [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2^2)^2}\right]$ [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P (X =x)	:	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

Find k , p (X < 5) , P (X > 5) [6]

.(c) If $v = 3x^2y + 6xy - y^3$,show that v is harmonic function

And find the corresponding analytic function . [8]
